Problèmes inverses
L'approche stochastique
Quand le hasard aide la géophysique

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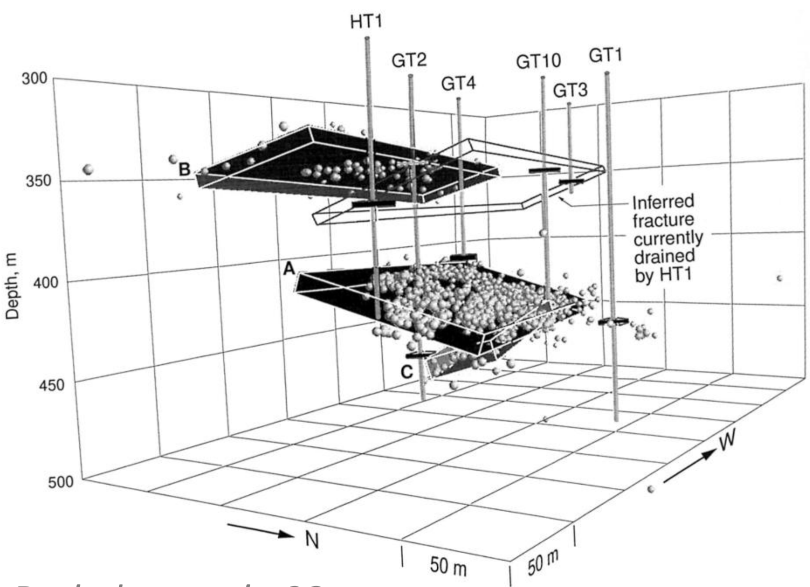
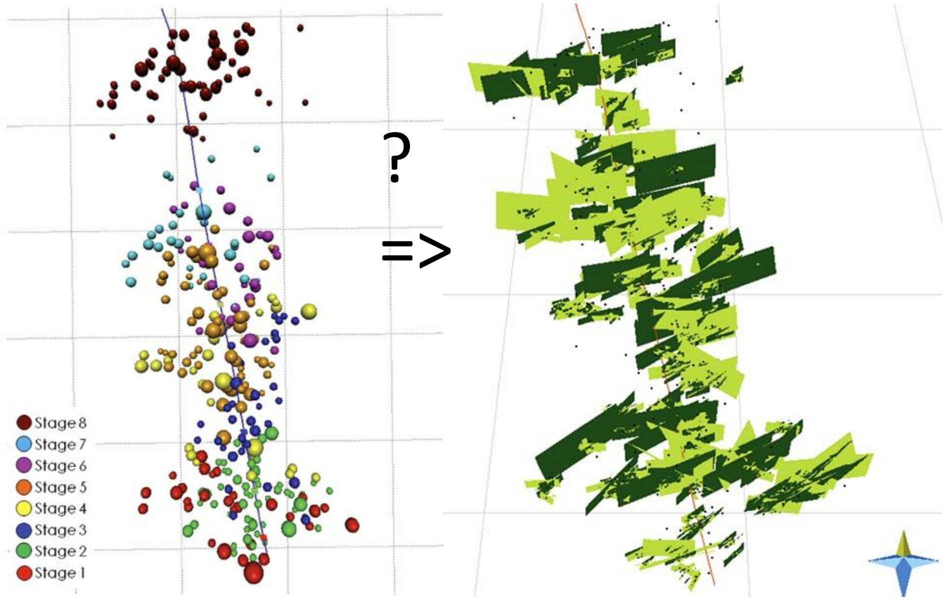
Contexte

- Contexte du monitoring microsismique



Rutledge et al., 98



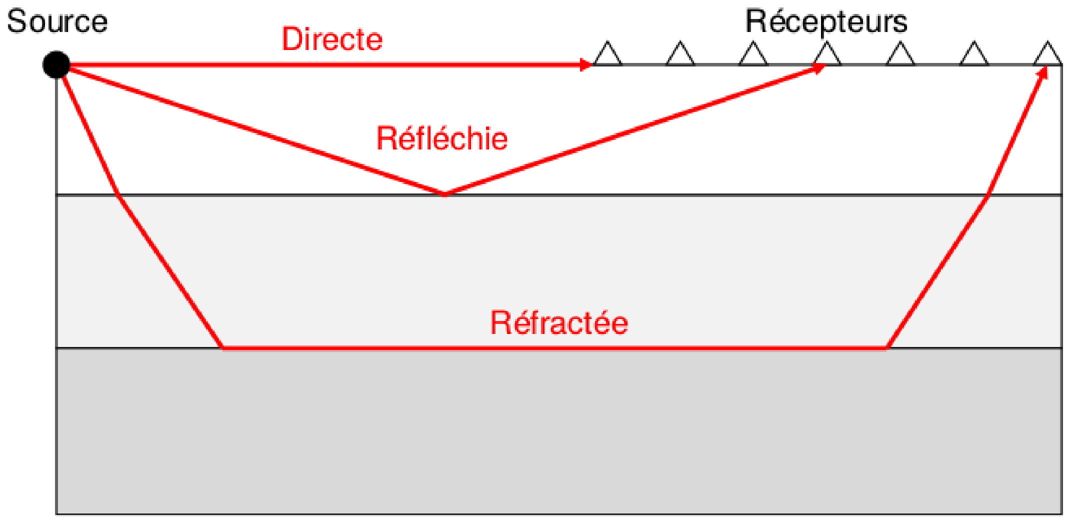
Williams-Stroud et al., 2013

Plans de fractures définis par les clusters d'hypocentres ou erreurs de localisation?

- Erreurs de localisation majoritairement dues à l'imprécision du modèle de vitesse
- => Importance d'avoir un modèle de vitesse fiable avec les incertitudes associées

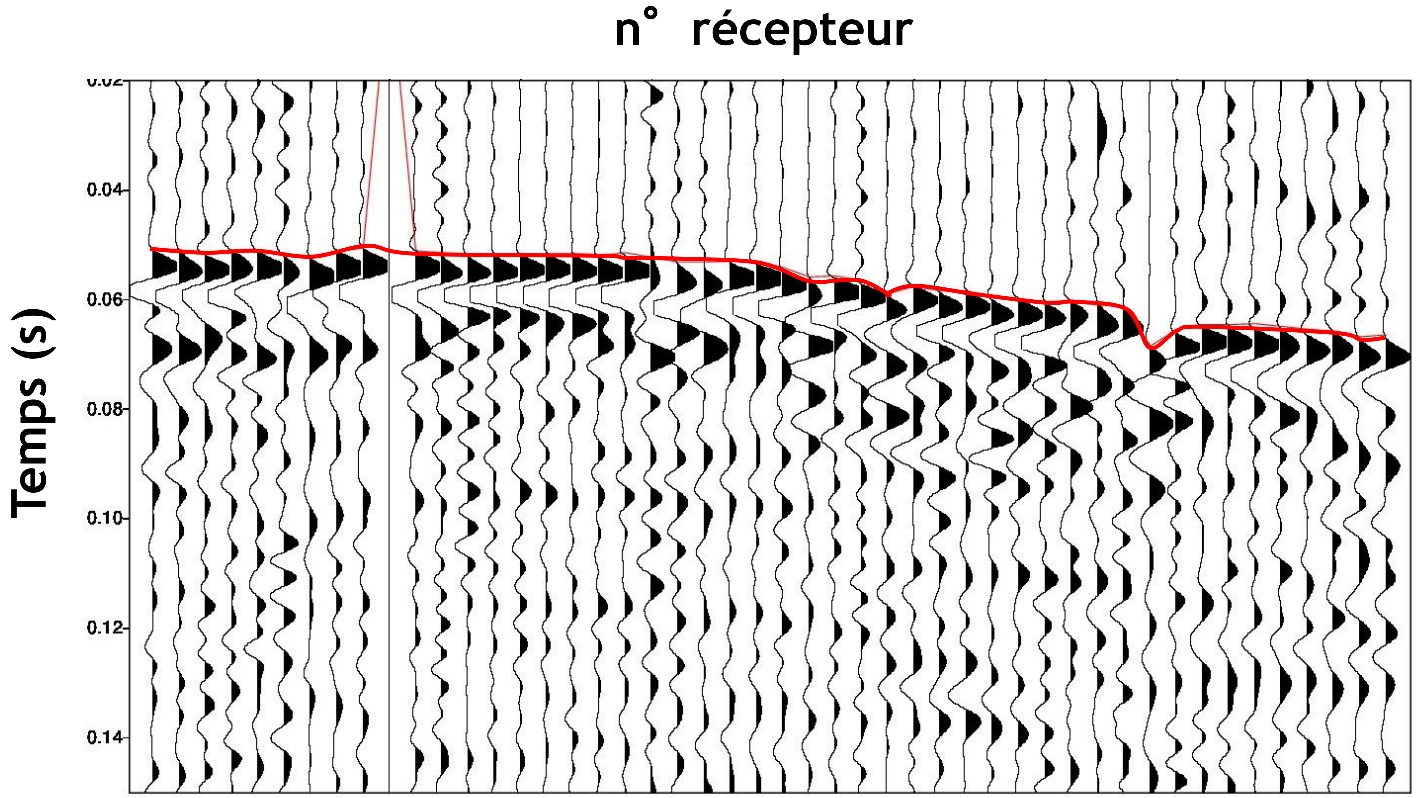
Tomographie sismique

Acquisition sismique



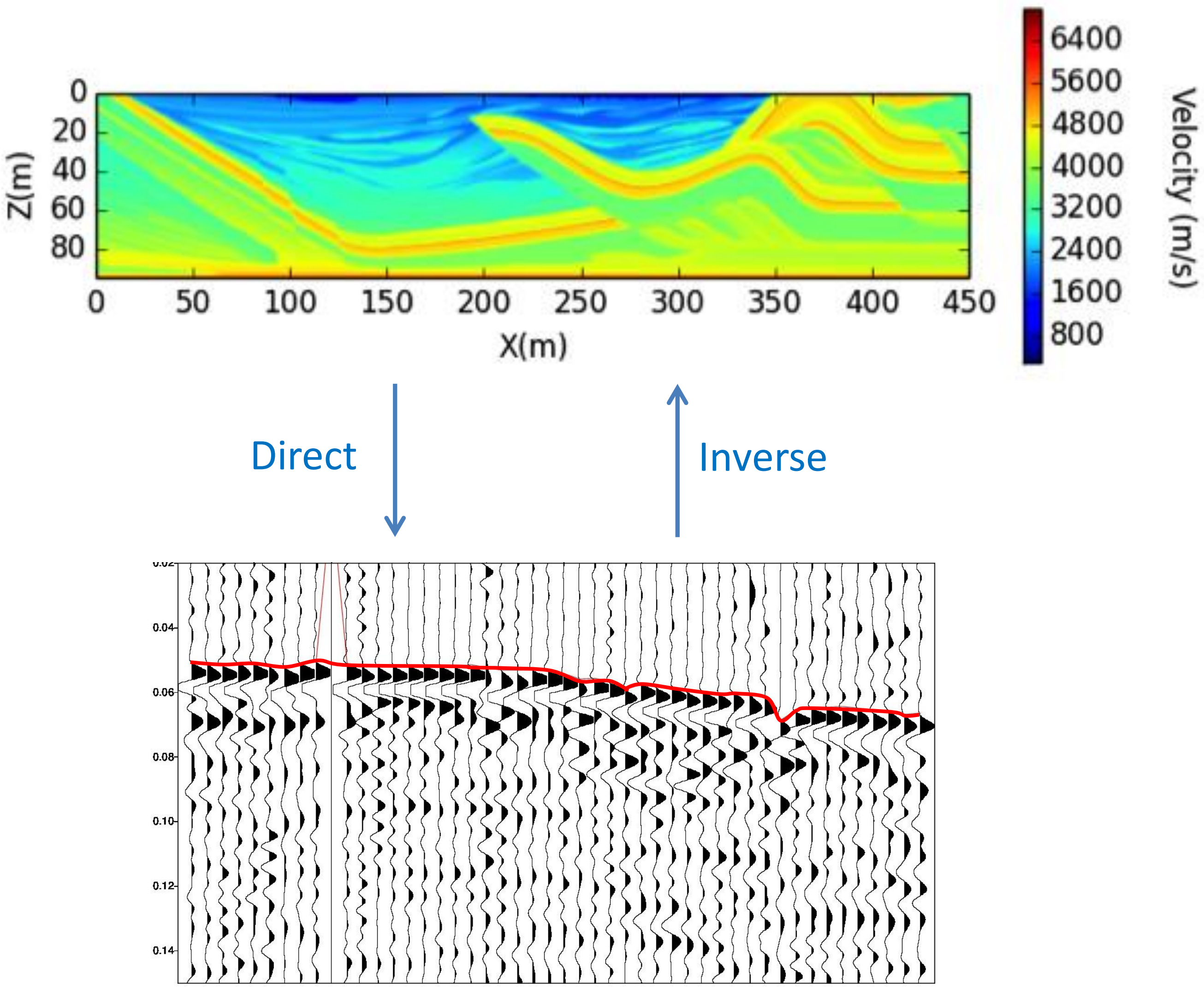
Tomographie sismique

○Observations : Temps d'arrivée pointés



Tomographie sismique

○Problème inverse



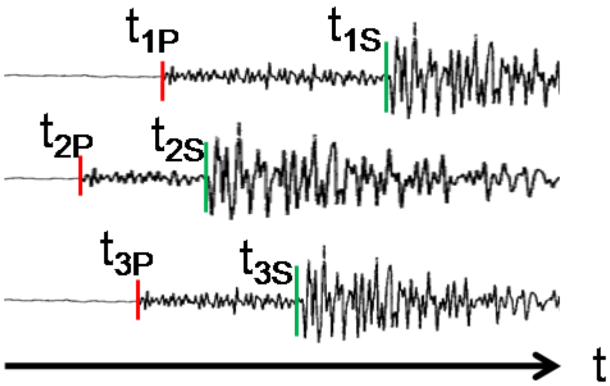
Tomographie sismique

○Problème inverse

- Problème d'optimisation numérique visant à minimiser une fonction coût qui mesure la qualité du modèle

$$E(\mathbf{m}) = (G(\mathbf{m}) - \mathbf{d_{obs}})^T \mathbf{C_d} (G(\mathbf{m}) - \mathbf{d_{obs}})$$

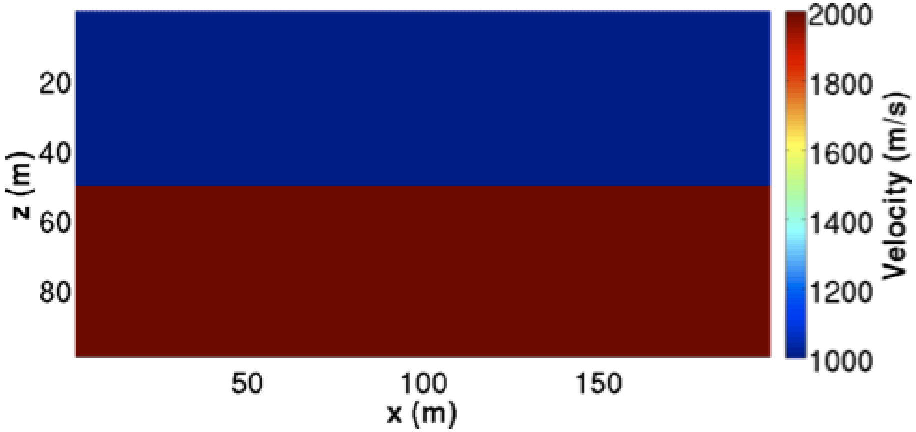
- $\mathbf{d_{obs}}$: données observées (temps d'arrivée pointés)
- **G(m)** : données synthétiques générées pour le modèle m avec **G**, l'opérateur du problème direct
- $\mathbf{C_D}$: matrice de covariance qui tient compte des erreurs de mesure



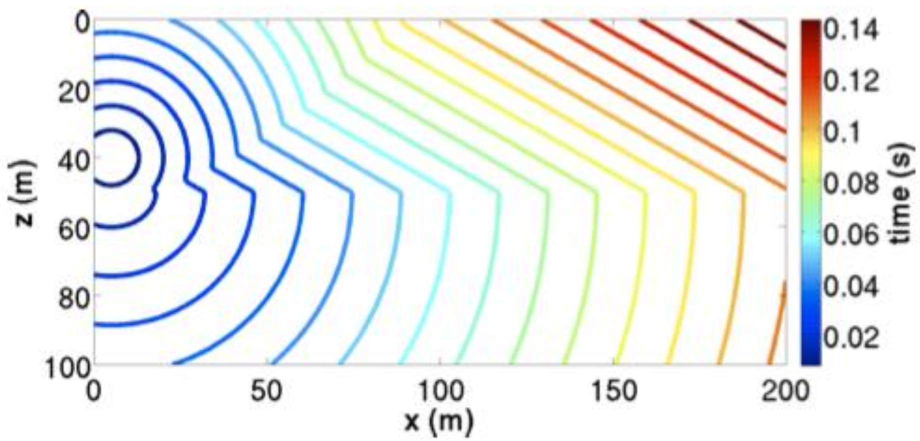
Tomographie sismique

○Problème direct

- Calcul des temps d'arrivée entre sources et récepteurs pour un ensemble de valeurs des paramètres du modèle.



$$\begin{pmatrix} t_{cal}^{1} \\ t_{cal}^{2} \\ \vdots \\ t_{cal}^{N} \end{pmatrix} = G \begin{pmatrix} V_p(1) \\ V_P(2) \\ z \end{pmatrix}$$



Tomographie sismique

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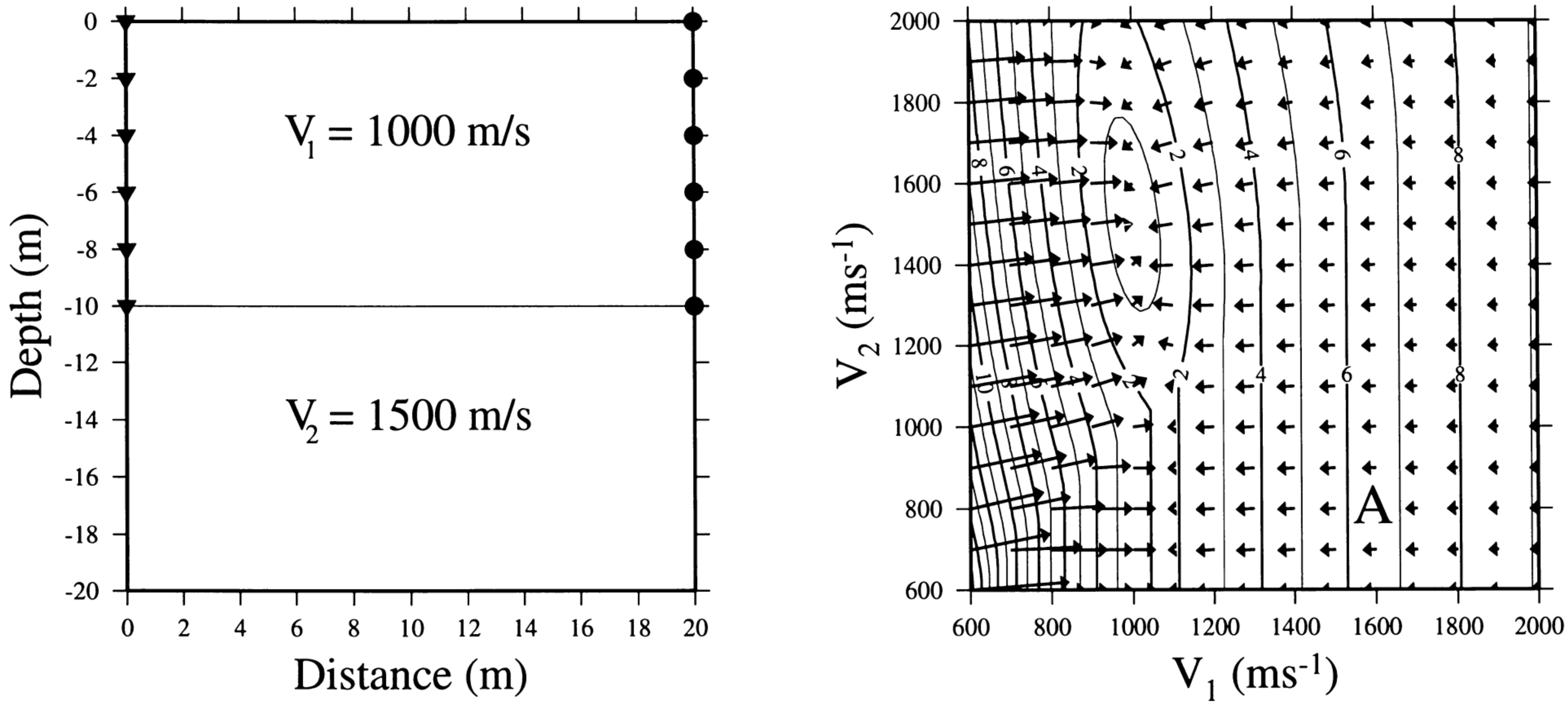
- Pour résoudre le probème inverse:
  - Techniques d'optimisation locale
  - Techniques d'optimisation globale

Tomographie sismique

Inconvénients des méthodes d'optimisation locales

- Très dépendantes du modèle initial



D'après Wéber, 2000

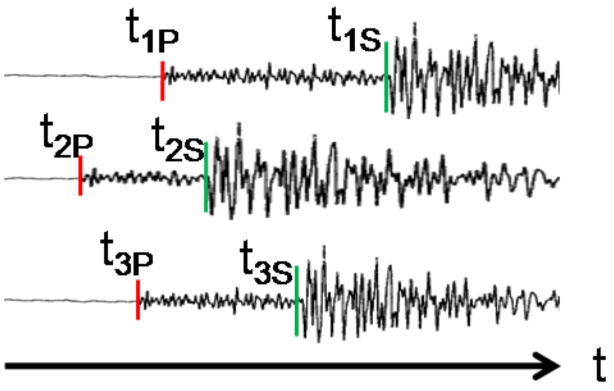
- Ne permettent pas d'estimer les incertitudes de manière fiable

Tomographie stochastique

- Objectif: Tomographie des temps de trajet avec estimation des incertitudes

- La tomographie sismique est souvent un problème inverse mal posé
  - Requiert d'utiliser toute l'information a priori disponible sur le modèle de vitesse
  - Requiert de représenter correctement les incertitudes sur les données

- Approche probabiliste adaptée (e.g. Tarantola & Valette, 82) :
  - Toute les informations sont représentées par des distributions de probabilité
  - On cherche à estimer la fonction de densité de probabilité des modèles sachant les temps d'arrivée observés $P(\mathbf{m}|\mathbf{d}_{obs})$



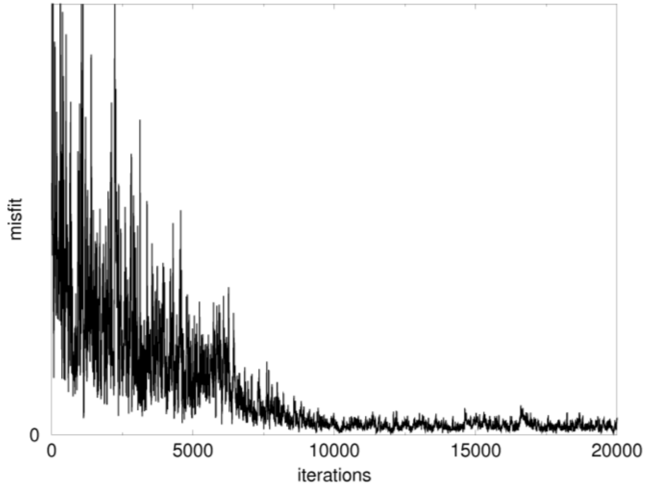
Tomographie stochastique

- Approche probabiliste adaptée : Distribution a posteriori consistante avec l'information a priori et les incertitudes sur les pointés

$$P(\mathbf{m}|\mathbf{d}_{\mathbf{obs}}) \propto P(\mathbf{m})\exp\left(-\frac{1}{2}\left\|\frac{G(\mathbf{m})-\mathbf{d}_{\mathbf{obs}}}{\sigma^d}\right\|^2\right)$$

- Estimation d'incertitudes : Echantillonnage de la distribution a posteriori avec un algorithme type Markov Chain Monte Carlo (MCMC) qui génère une chaîne aléatoire de modèles dont la distribution approxime P($\mathbf{m}|\mathbf{d}_{\mathbf{obs}}$)

- => Requiert:
  - ❑ Des algorithmes efficaces
  - ❑ Des paramétrisations parcimonieuses



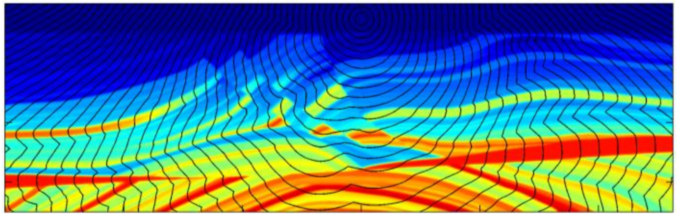
Forward problem

- Efficient and accurate Eikonal solver to compute first arrival traveltimes in complex 3D media

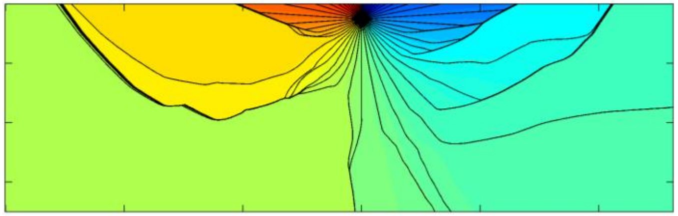
$$\left(\frac{\partial t}{\partial x}\right)^2 + \left(\frac{\partial t}{\partial y}\right)^2 + \left(\frac{\partial t}{\partial z}\right)^2 = s^2$$

  - Spherical wave approximation close to the source
  - Plane wave approximation far from the source

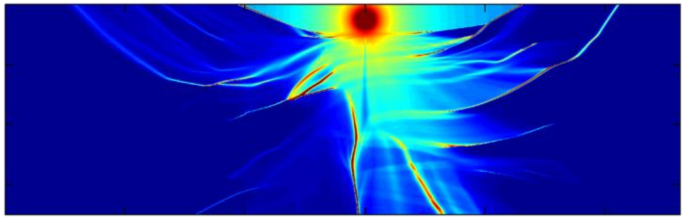
- Allows to compute auxiliary quantities such as take-off angles and amplitudes



Traveltimes



Take-off angles



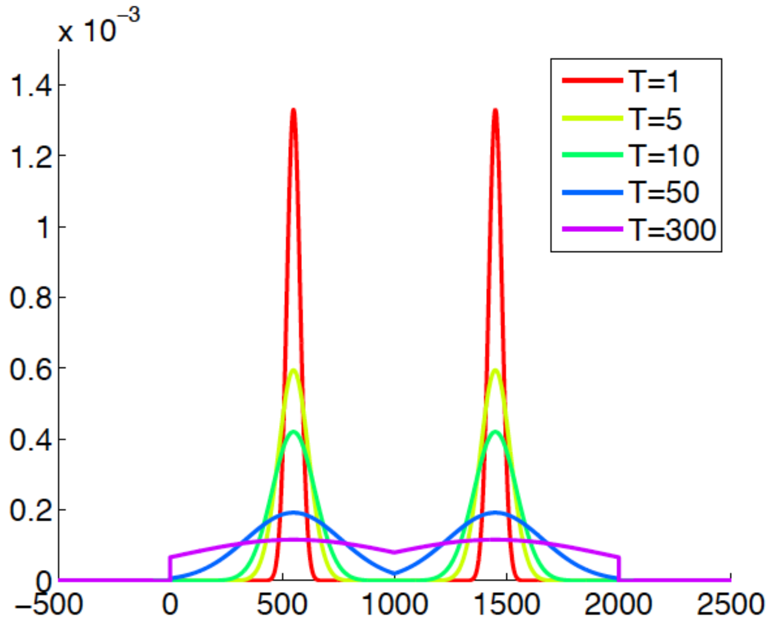
Amplitudes

Noble, Gesret and Belayouni, 2014

Simulated Annealing algorithm

- Classical Monte-Carlo sampling of the tempered a posteriori distribution

$$P_T(m|d_{obs}) \propto \exp\left(-\frac{E(m)}{T}\right)$$



T decreases during the run. At T=1:

$$P_T(m|d_{obs}) = P(m|d_{obs})$$

- A single chain can fail to sample the whole distribution by getting trapped in one local mode.

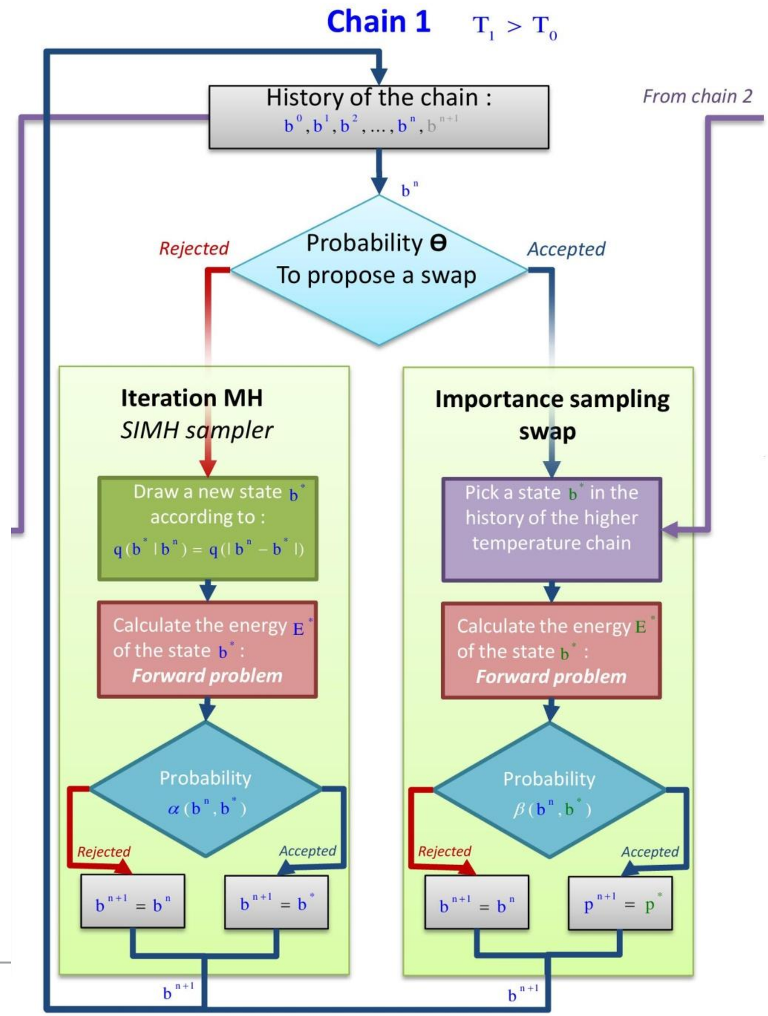
Interacting Markov chains

- Allowing several Markov chains at different temperatures to interact in order to sample very efficiently the a posteriori distribution (Geyer, 91)

- **Parallel tempering**: exchanges between simultaneous states of chains at non adjacent temperatures

- **IR-MCMC**: Exchanges in the history of the chain at higher adjacent temperature

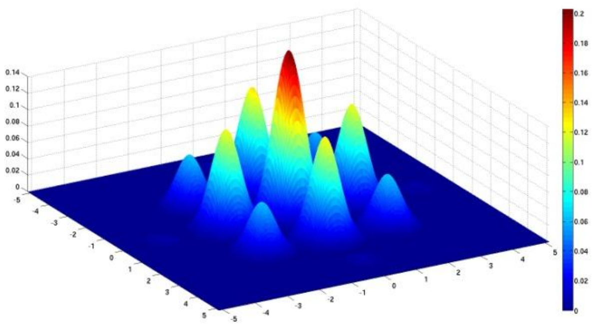
$\Rightarrow$ Improves mixing properties and efficiency (Romary 2010)



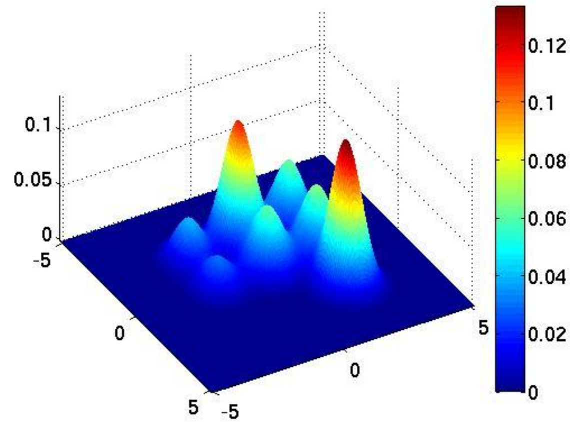
Interacting Markov chains

- Test on a highly multimodal distribution

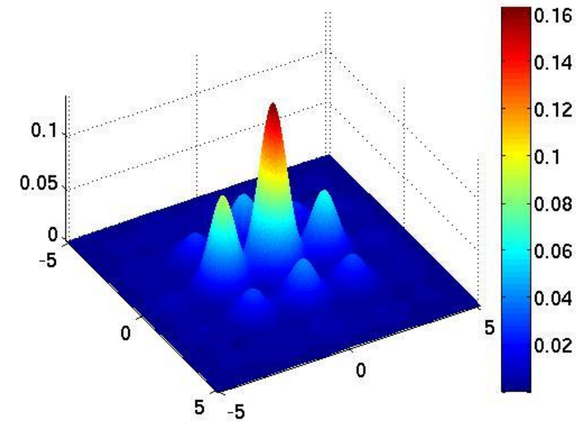
$P(\mathbf{m}|\mathbf{d}_{\mathrm{obs}})$



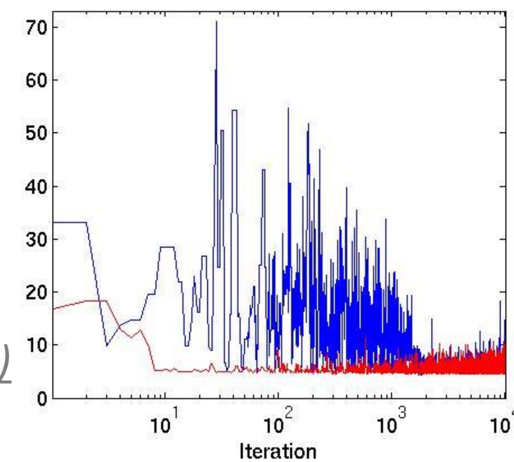
Distribution by SA



Distribution by IR-MCMC



- IR-MCMC algorithm
  - ❑All the modes are identified
  - ❑Global mode identified as the principal one
  - ❑Much more efficient

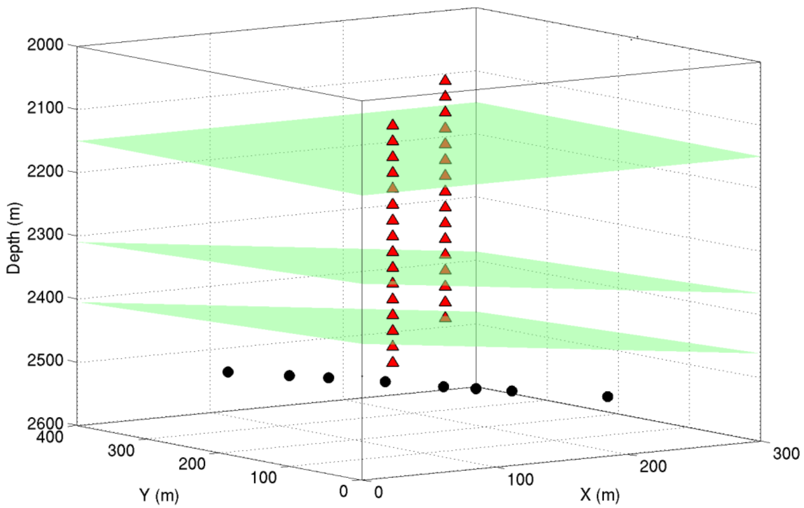


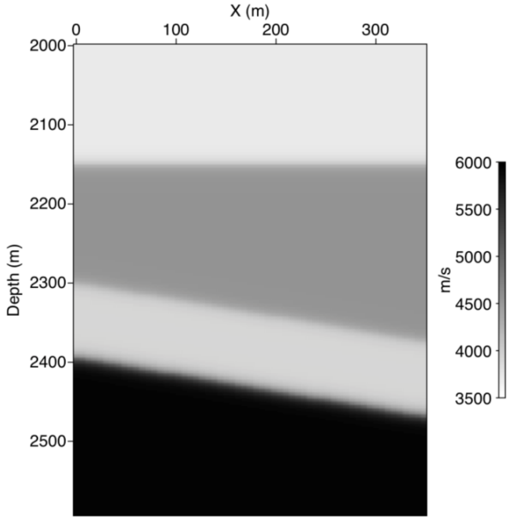
Convergence for lower temperature chain (red) and for Simulated Annealing (blue)/Log scale

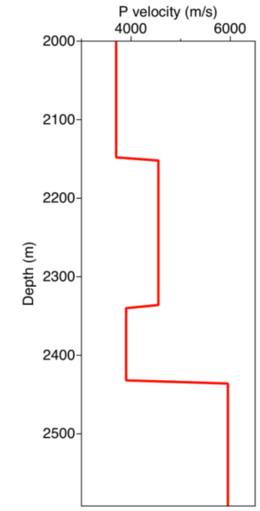
Bottero et. al., 2016

Stochastic tomography : synthetic case

- Microseismic context







▲ receivers

● calibration shots

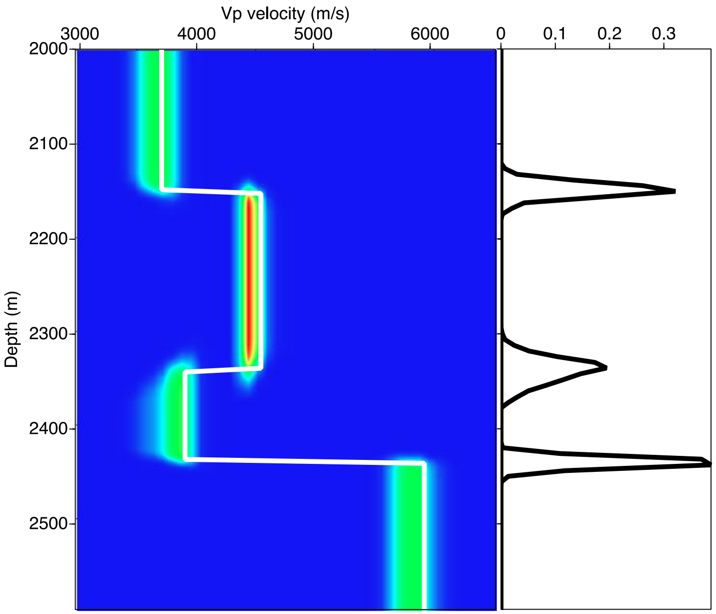
$$P(\mathbf{m}|\mathbf{t_{obs}}) \propto \exp\left(-\frac{1}{2}\sum_{j=1}^{nshots}\sum_{i=1}^{nsensors}\left(\frac{t_{obs}^{j,i}-t_{cal}^{j,i}(\mathbf{m})}{\sigma_{t_{obs}}^{j,i}}\right)^2\right)$$

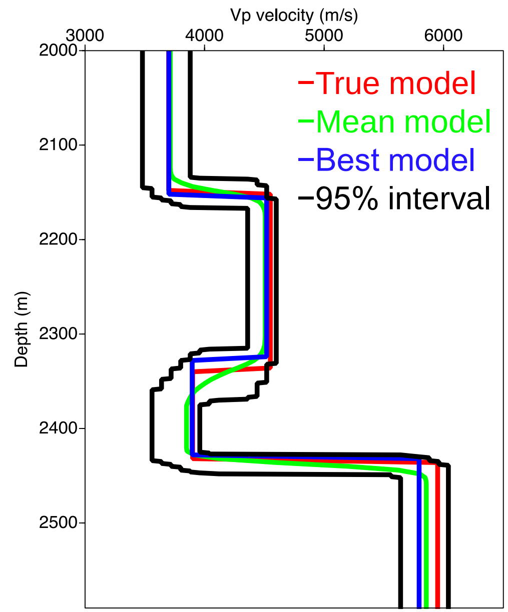
- 4 layer P and S velocity models (13 parameters)
- Traveltimes computed with an accurate Eikonal solver (Noble et. al., 2014)
- Gaussian noise added to the observed traveltimes

Stochastic tomography : results

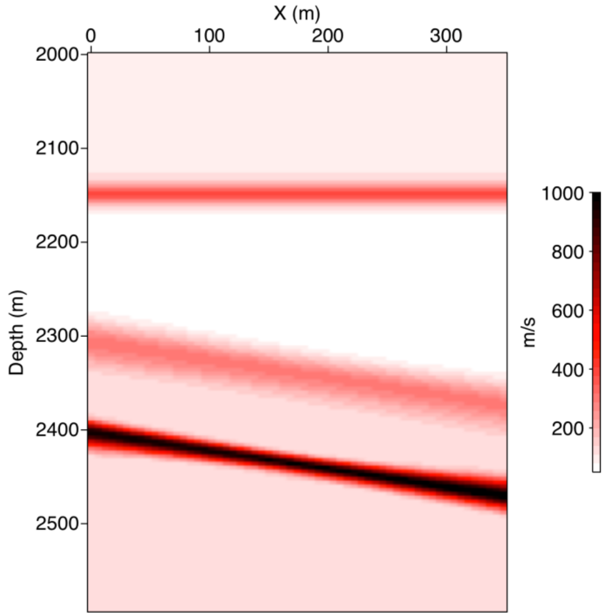
- Large number of models consistent with a priori information and data uncertainties

Posterior PDF for Vp and z



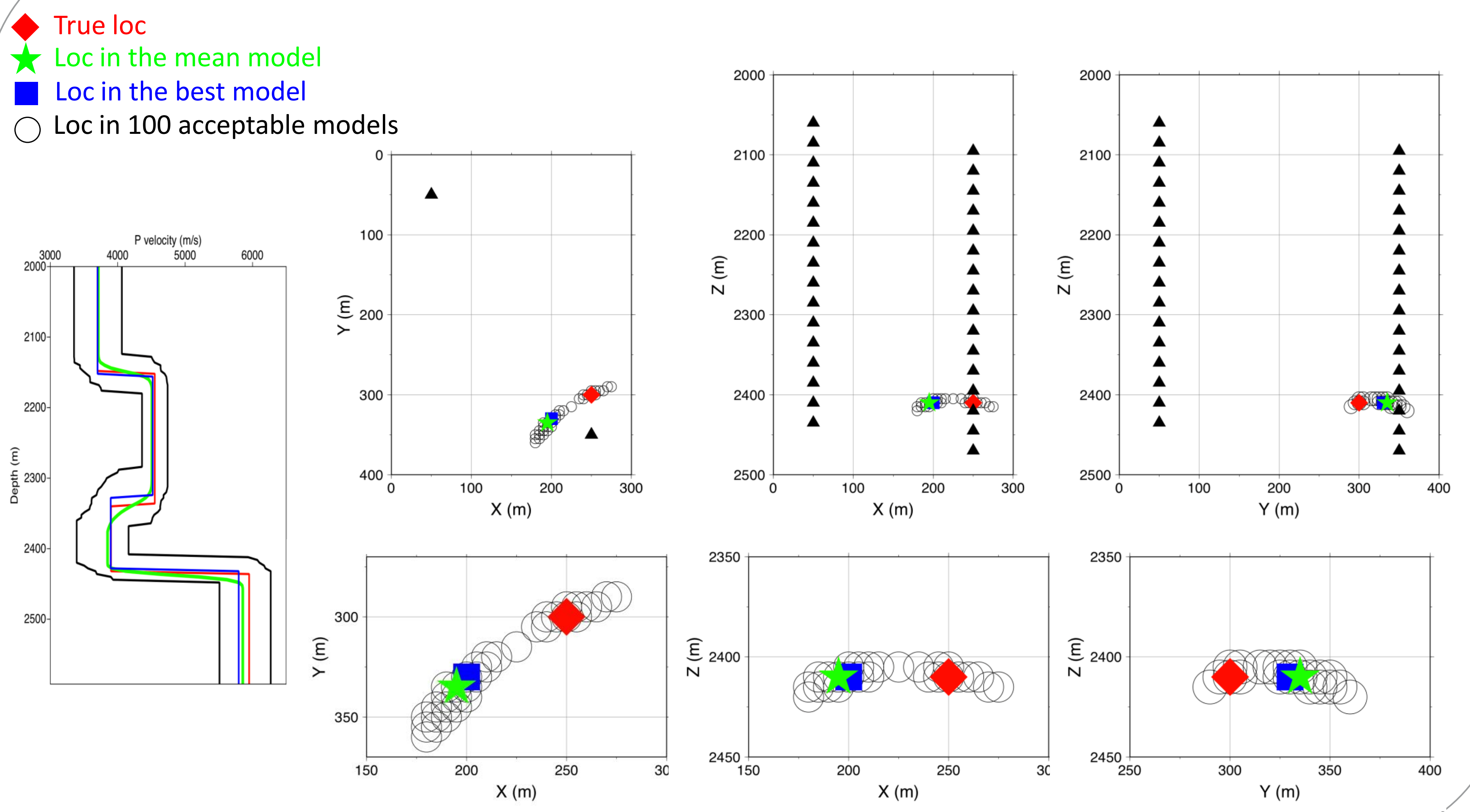


Standard deviation



- Similarity between the true and the mean model
- Identification of the well and poorly constrained regions
- True estimation of the uncertainties

Influence of velocity model on event location



Gesret et. al., 2015

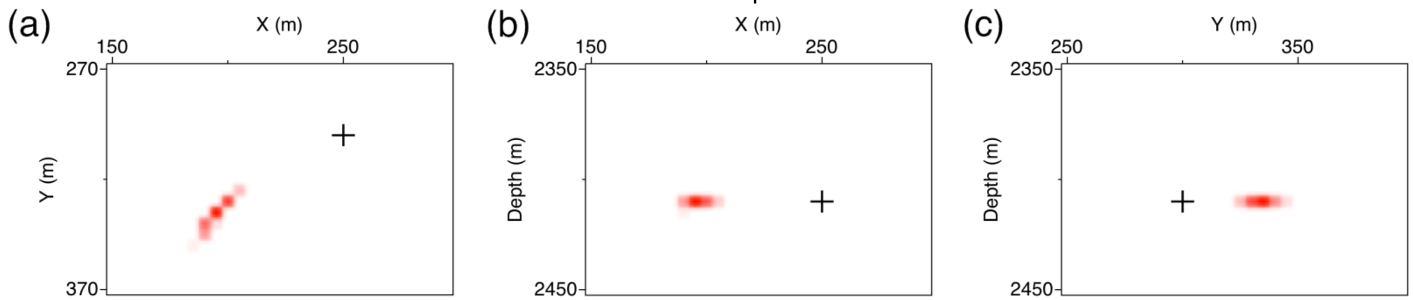
Probabilistic seismic event location

- Standard probabilistic location

$$P(\mathbf{l}|\mathbf{d}_\mathbf{s}^{\mathbf{obs}}, v) \propto \exp\left(-\frac{1}{2}\sum_{i=1}^{nsensors}\left(\frac{d_s^{obs,i} - d_s^{cal,i}(\mathbf{l}, v)}{\sigma_s^i}\right)^2\right)$$

<u>*68% confidence intervals built from the marginal pdfs*</u>

$P(x,y|d_s^{obs},v)$ $\quad$ $P(x,z|d_s^{obs},v)$ $\quad$ $P(y,z|d_s^{obs},v)$



$\Rightarrow$ Only one velocity model!

- Propagation of velocity uncertainties to event location

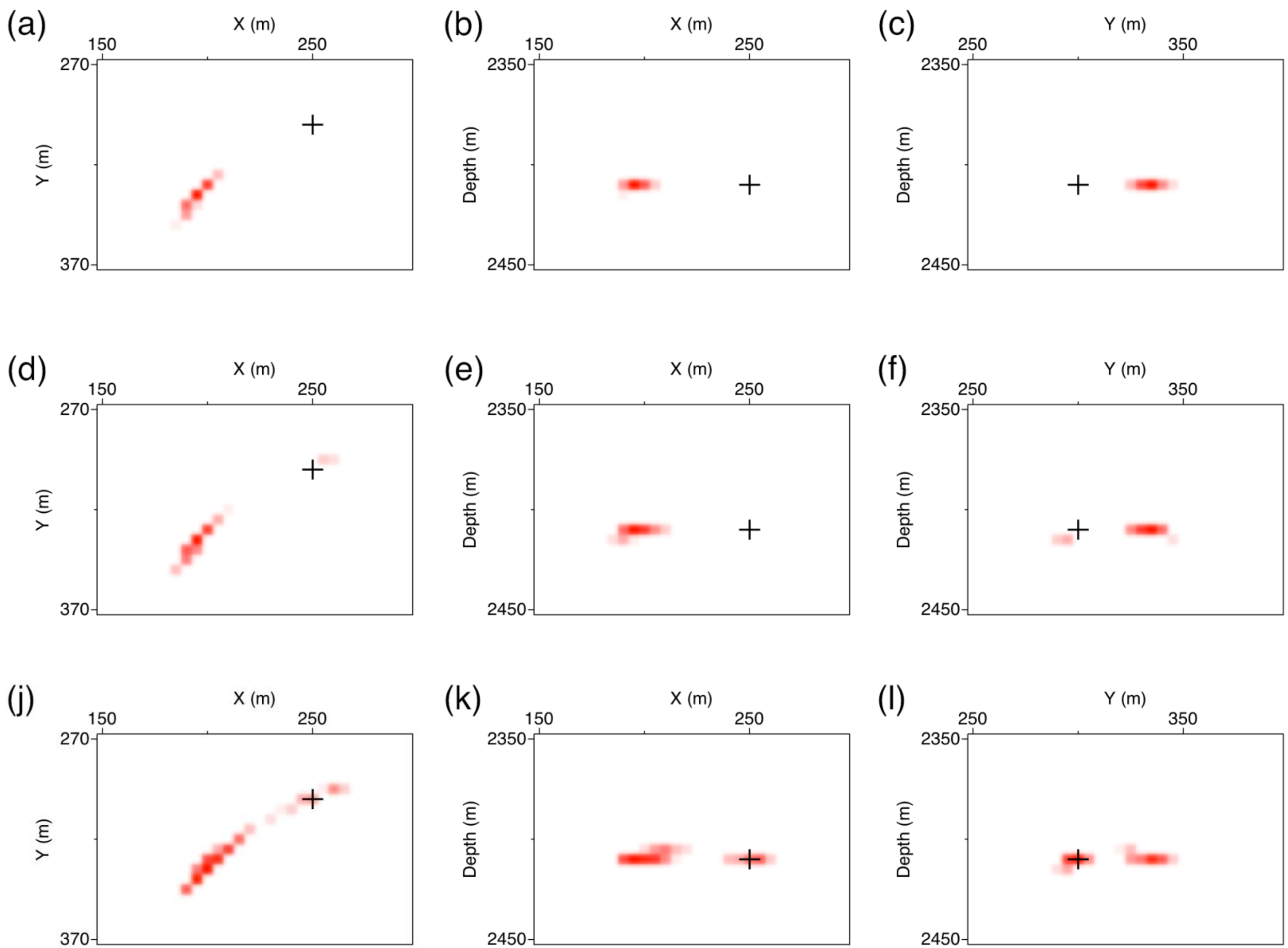
$$P(\mathbf{l}|\mathbf{d}_\mathbf{v}, \mathbf{d}_\mathbf{s}^{\mathbf{obs}}) \propto \frac{1}{N}\sum_{j=1}^{N}\exp\left(-\frac{1}{2}\sum_{i=1}^{nsensors}\left(\frac{d_s^{obs,i} - d_s^{cal,i}(\mathbf{l}, v_j)}{\sigma_s^i}\right)^2\right)$$

Standard formulation / New formulation

Location in the mean model

Location in the best model

New PDF

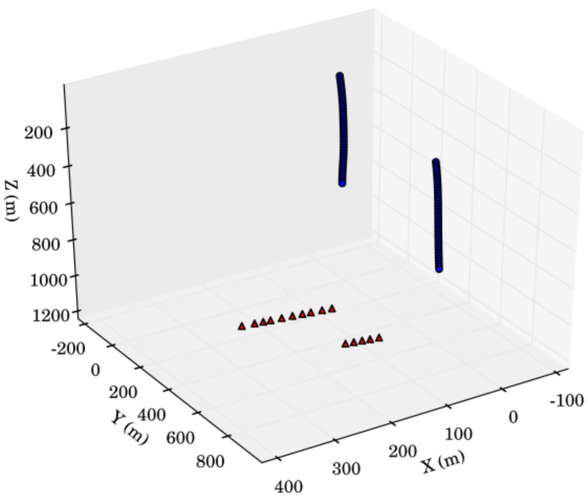


Gesret et. al., 2015

Application to real data

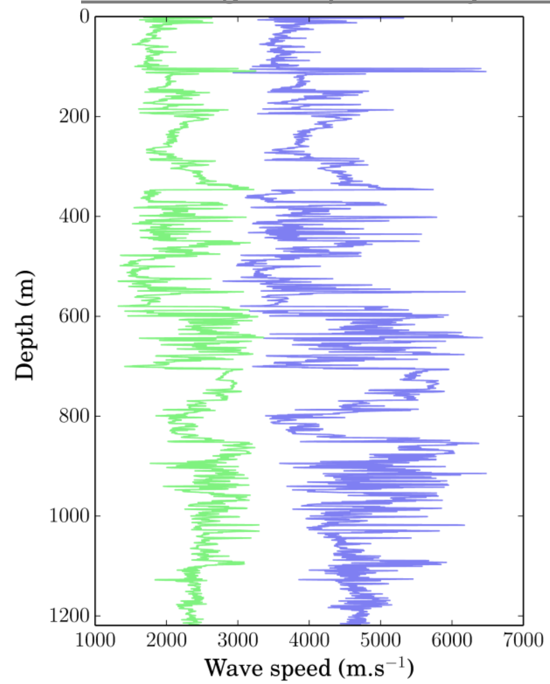
- Parameterization based on wavelet decomposition

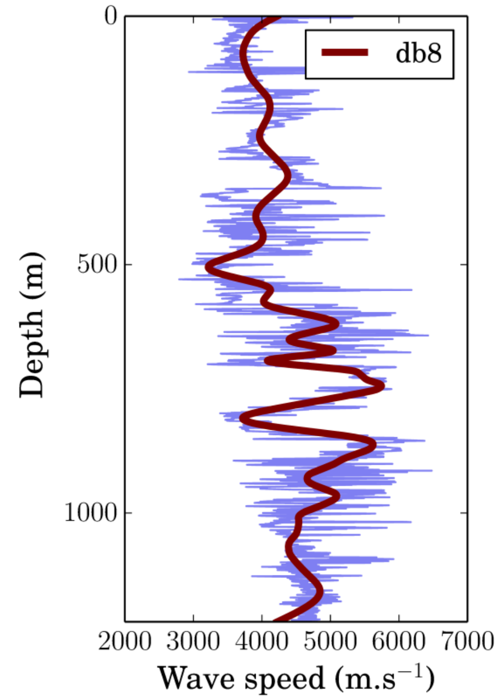
Acquisition geometry



▲ calibration shots
● receivers

Sonic logs : a priori information



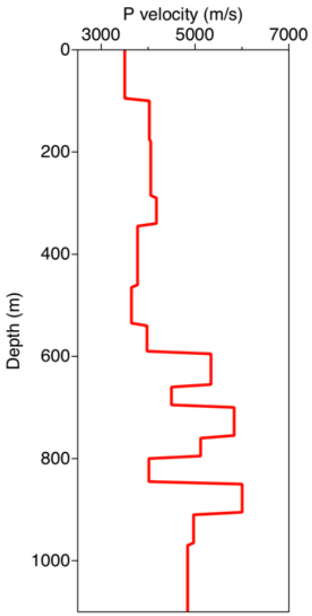


- Sonic logs decomposed into a basis of wavelet functions $V(x_j) = \sum_{i=1}^{N} b_i \psi_i(x_j)$
- Complex models described by a small number of coefficients b_i used as parameters of the inversion

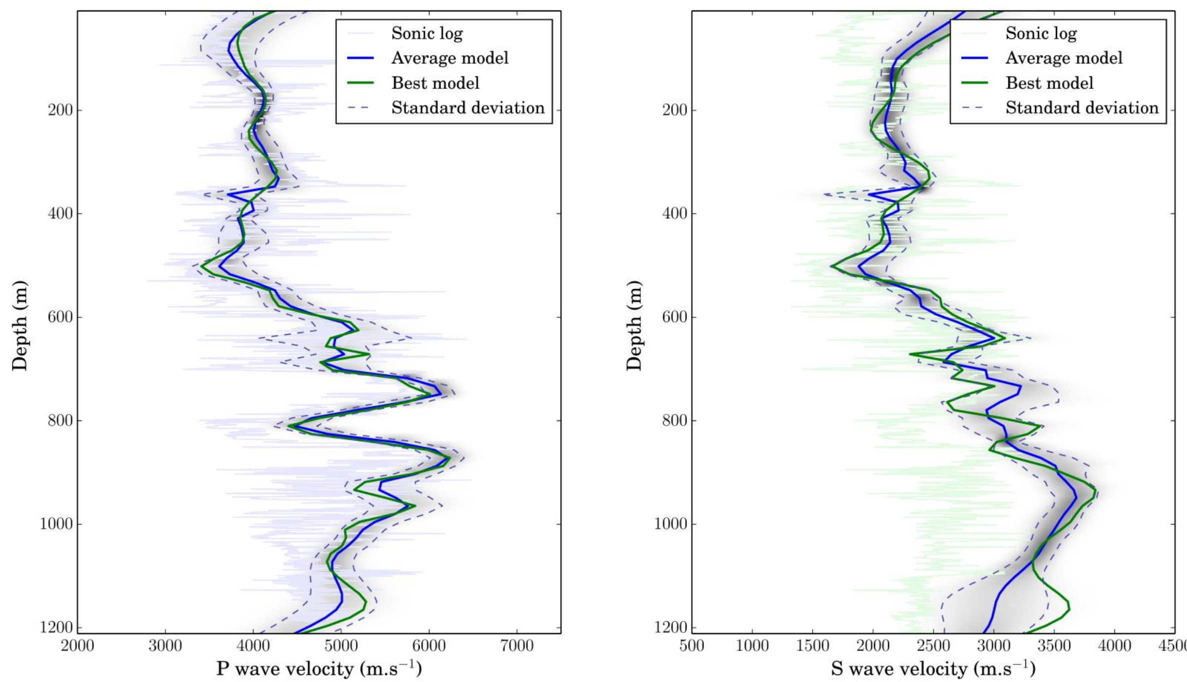
Bottero et. al., 2016

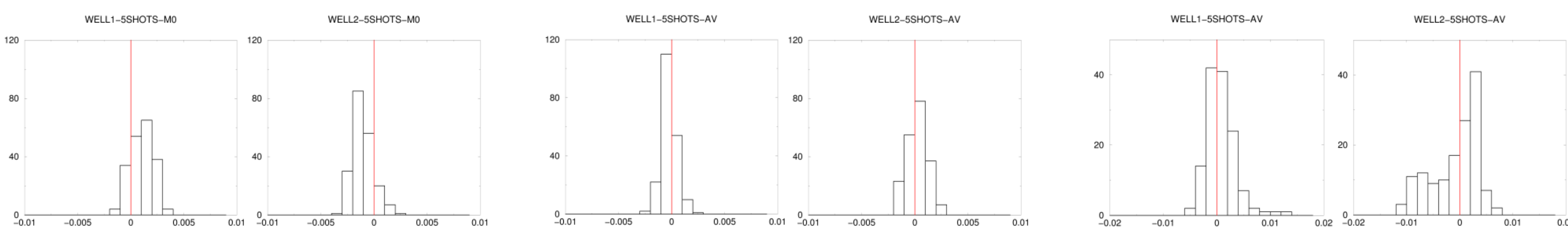
Application to real data

Reference model



Stochastic tomography results



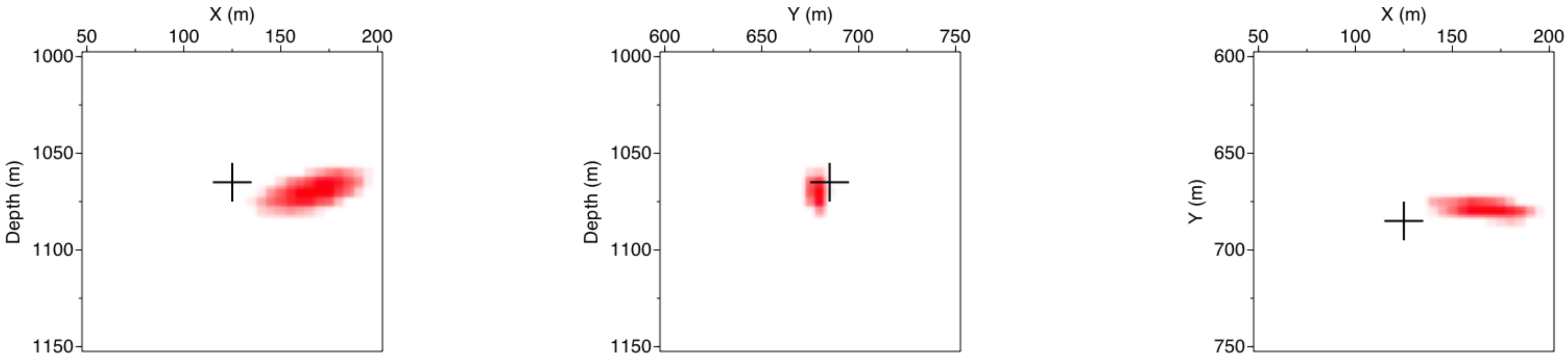


Bottero et. al., 2016

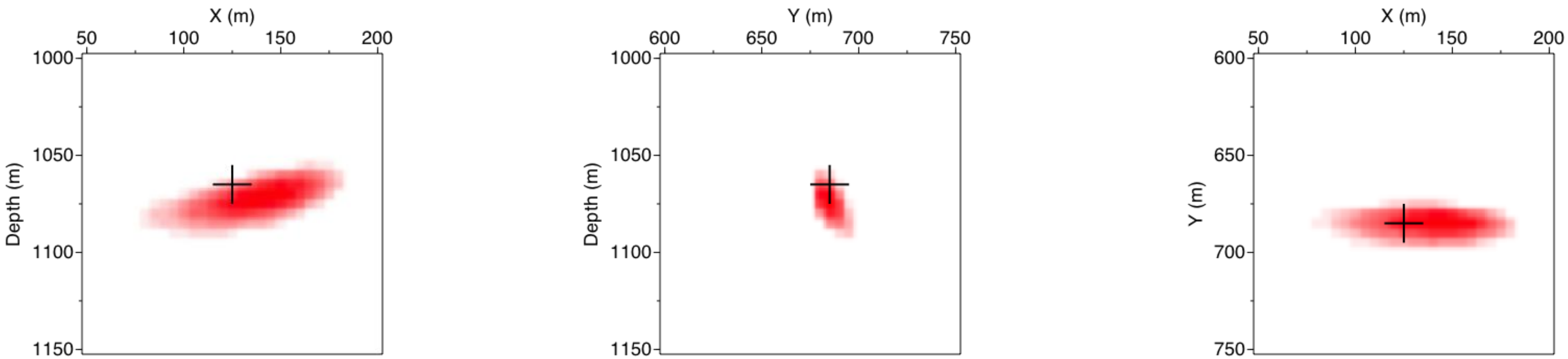
Application to real data

○Relocation of one calibration shot

Standard PDF for the reference velocity model



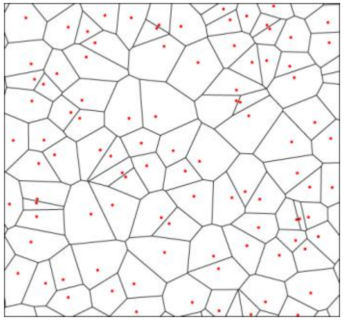
New PDF accounting for the velocity model uncertainties

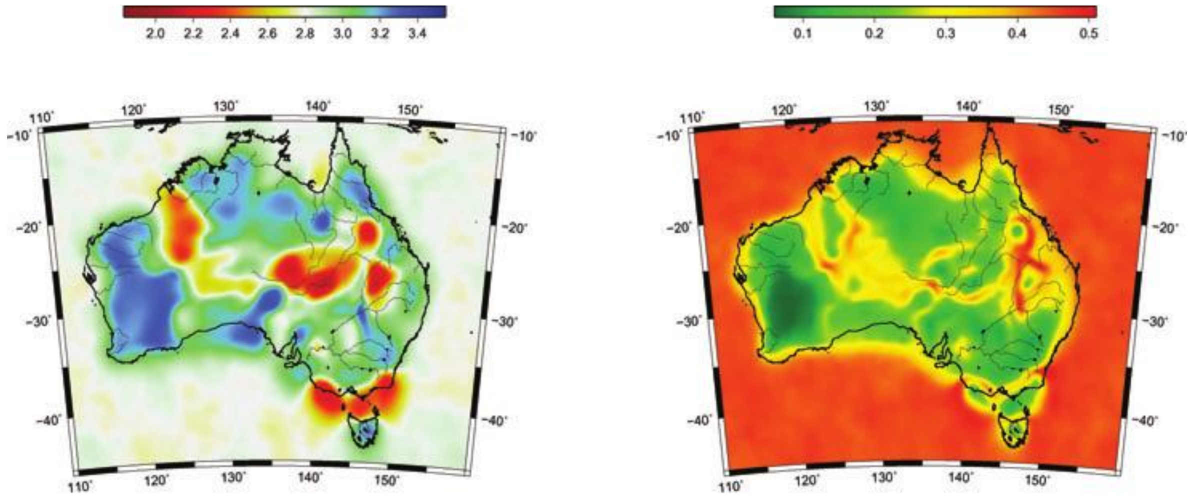


2D transdimensional tomography

- Number of model parameters = unknown of the inversion
- rj-MCMC algorithm (Green, 1995) : generalization of classical MH algorithm that allows simulation of the unknown space dimension

- Most common tessellation : Voronoi tessellation



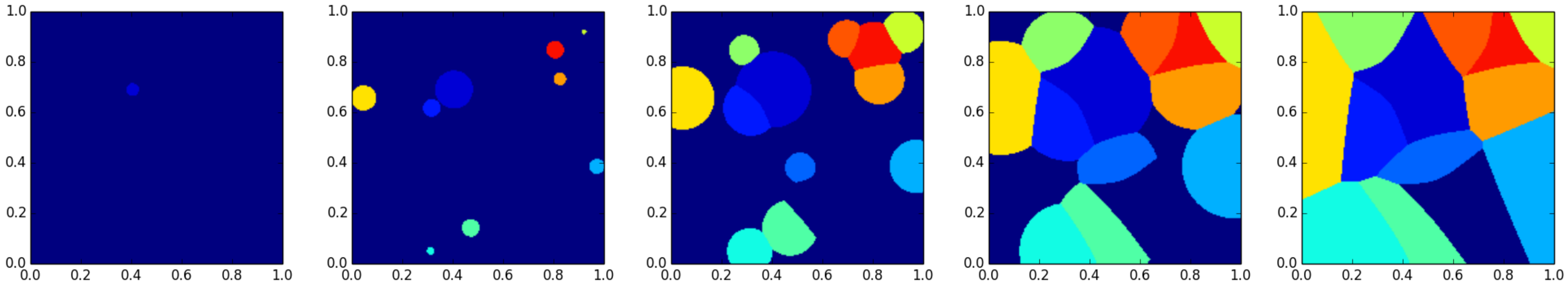


(a) Average solution map (km/s)   (b) Estimated error map (km/s).

Ambient noise tomography (From Bodin & Sambridge, 2009)

New parameterization : Johnson Mehl tesselation

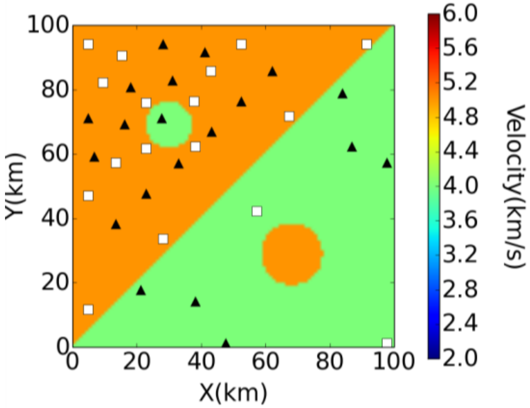
- Generalization of the Voronoi tesselation where the nuclei are increased additively with time
- All cells grow then isotropically with a constant speed around their nuclei. The growth of cell boundaries is stopped when they meet.



- Unlike Voronoi tessellation, Johnson-Mehl tessellation contains cells that have not necessarily flat sides.
- Model parameters: **m**=(U,v,t,n)
- rj-MCMC algorithm

Johnson Mehl tesselation

○Simple synthetic model (Bodin et. al., 2009)

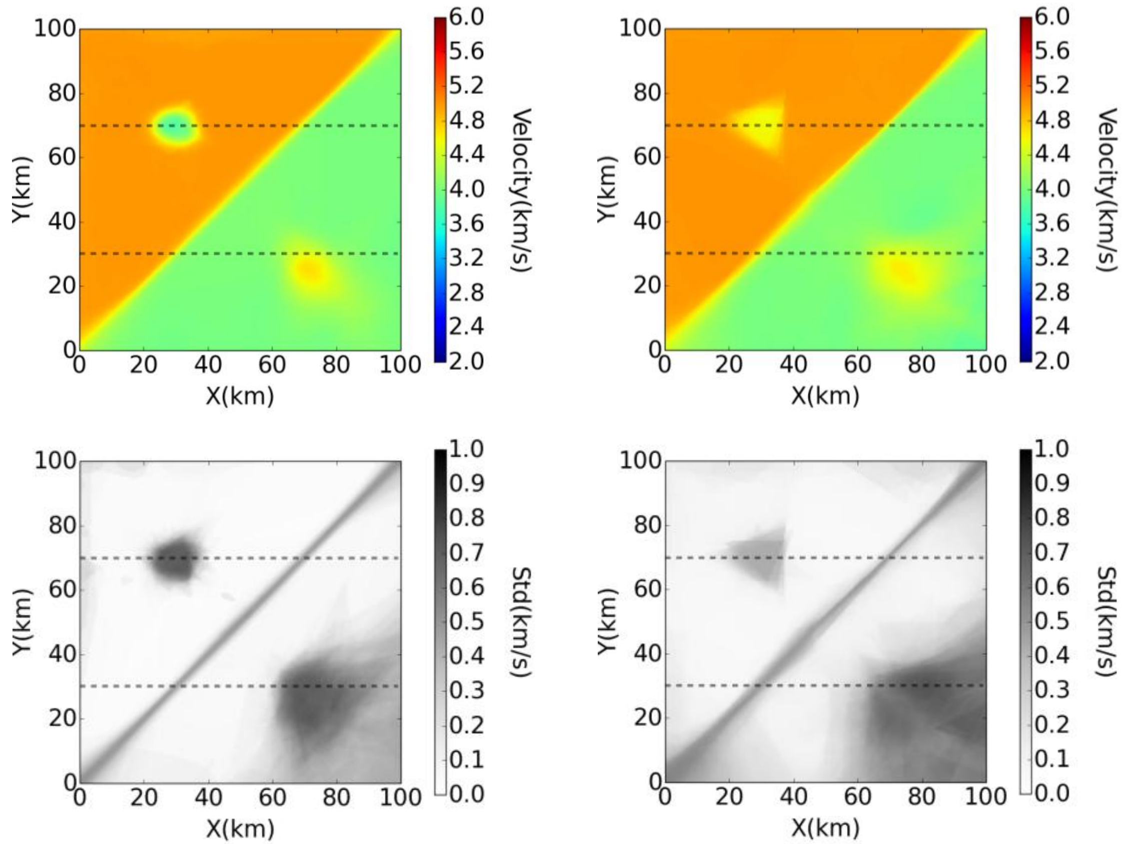


True model

17 sources
20 receivers

Posterior mean

Standard deviation



Johnson Mehl

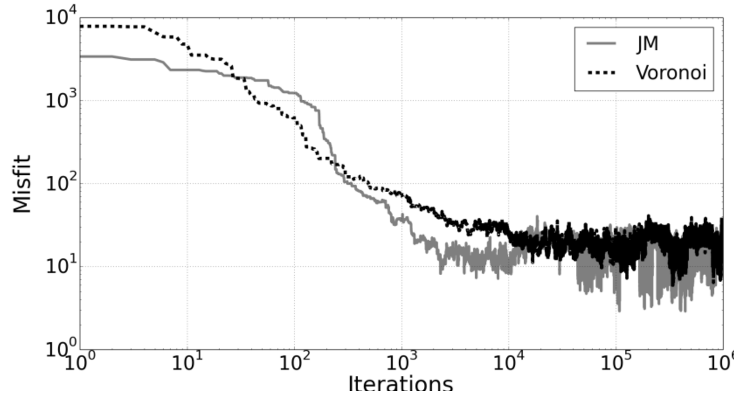
Voronoi

- Better results with JM tesselation

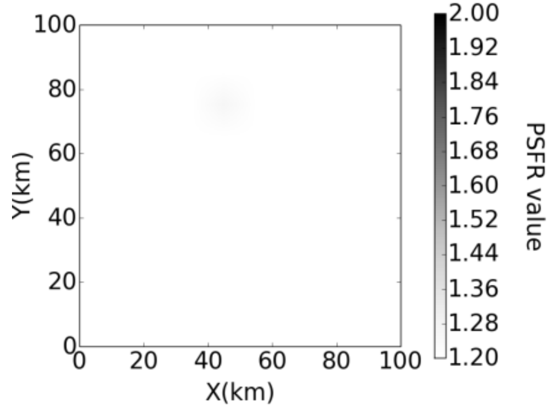
Johnson Mehl tesselation

○Simple synthetic model (Bodin et. al., 2009)

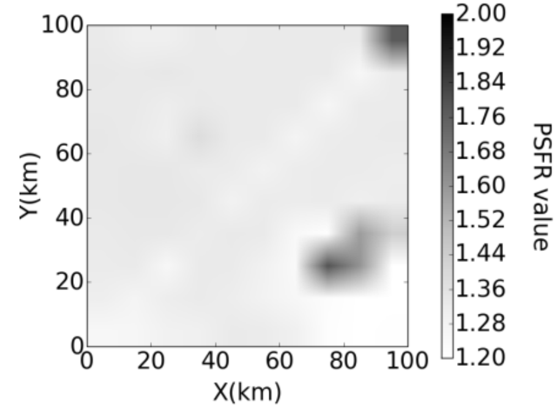
Evolution of the misfit



Gelman-Rubin diagnostic



Johnson Mehl



Voronoi

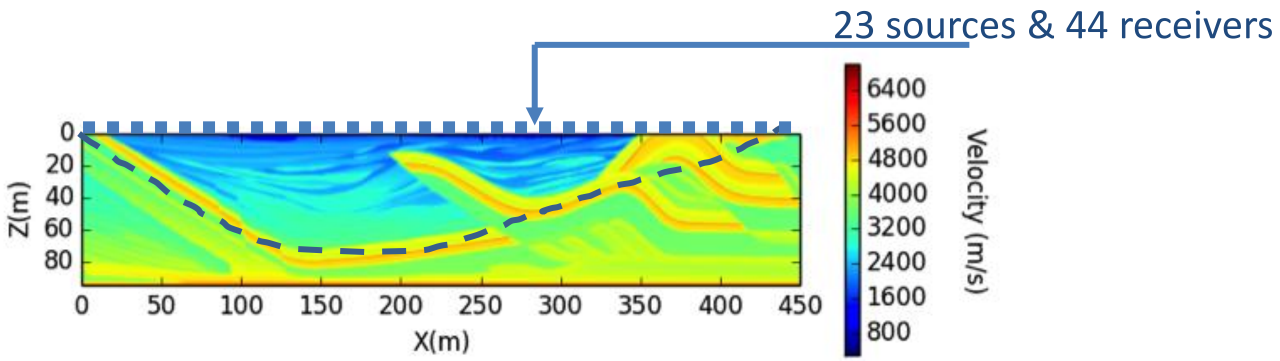
- Improvement of efficiency

Belhadj et. al., 2018

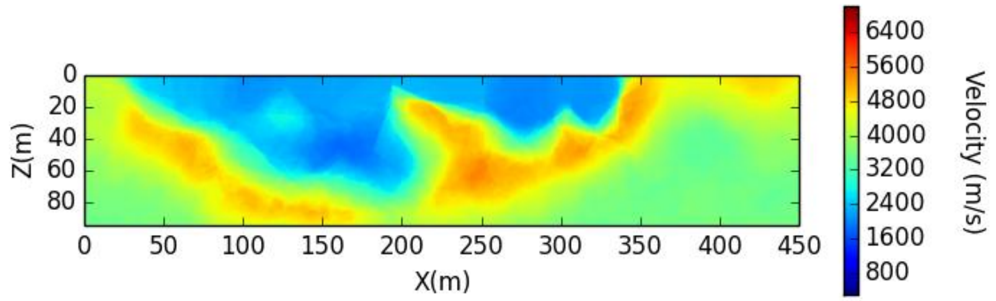
Johnson Mehl tesselation

- Benchmark model developed at Amoco Tulsa Research Lab

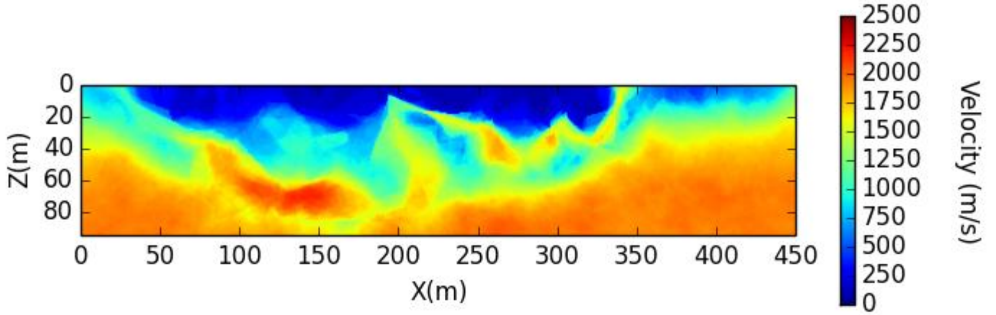
True model



Posterior mean value



Standard deviation



Belhadj et. al., 2018

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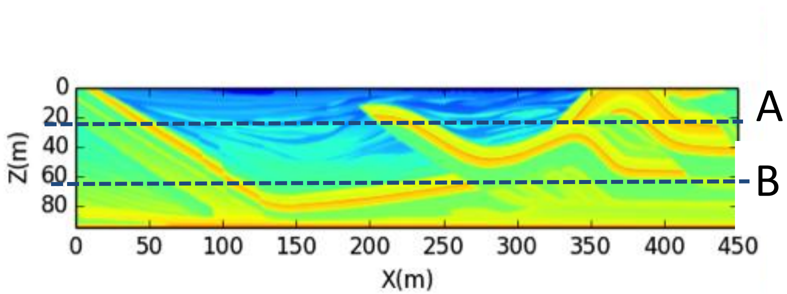

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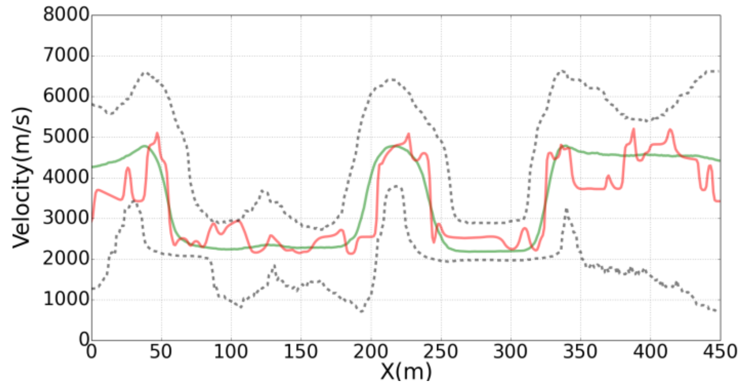
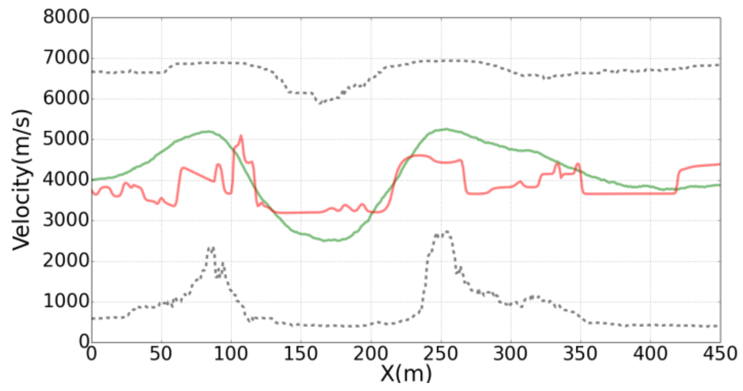


B



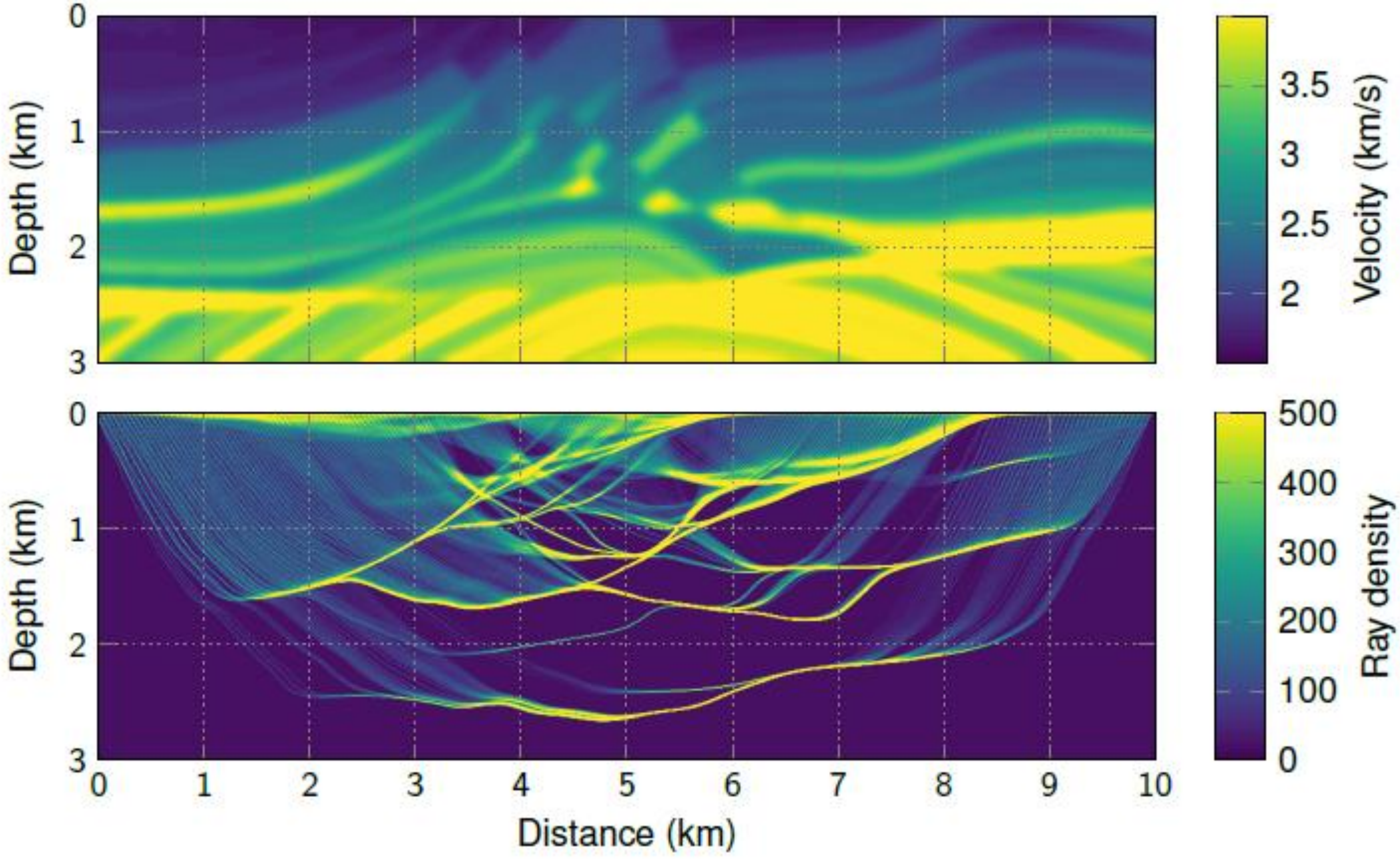
- True model
- Posterior mean model
- 95% confidence interval

Belhadj et. al., 2018

Competitive Particle Swarm Optimization

○Marmousi velocity model

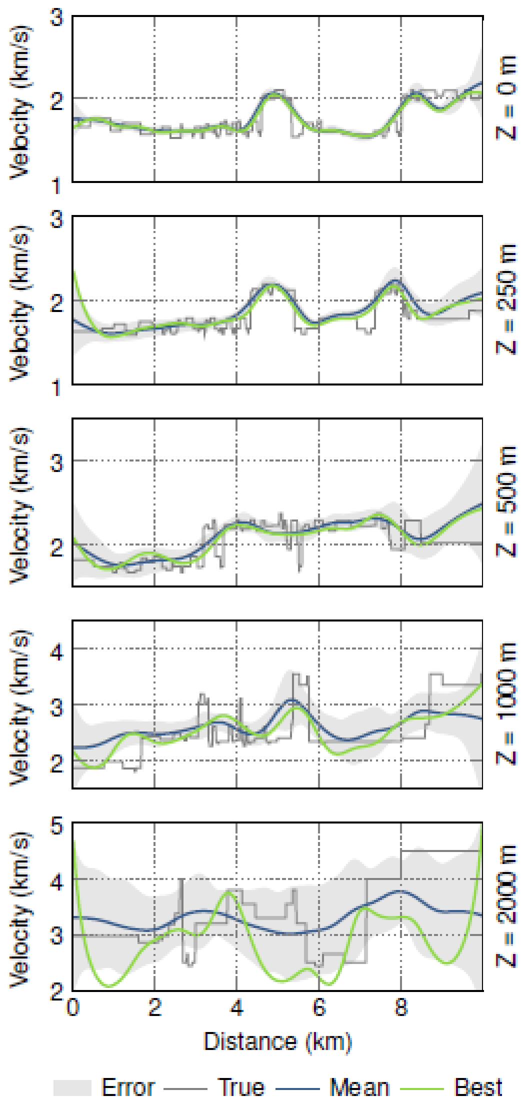
200 sources
400 receivers

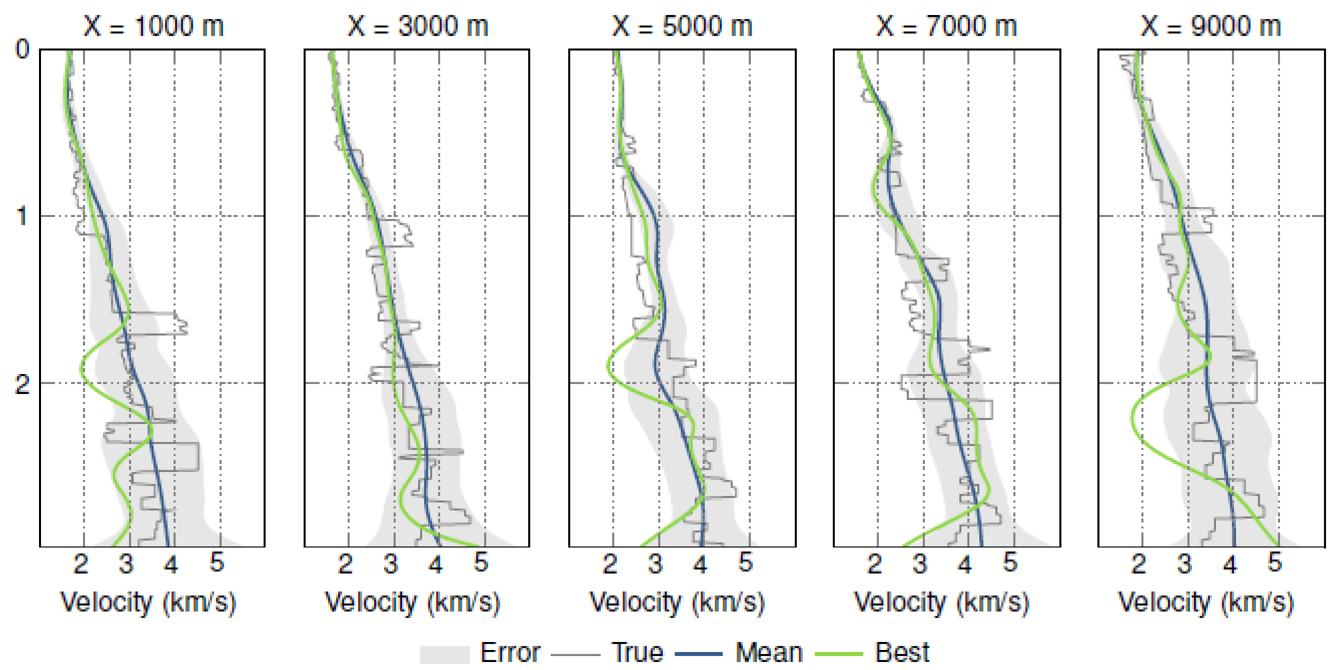


Luu et. al., submitted to Geophys. Prosp.

Competitive Particle Swarm Optimization

- Results



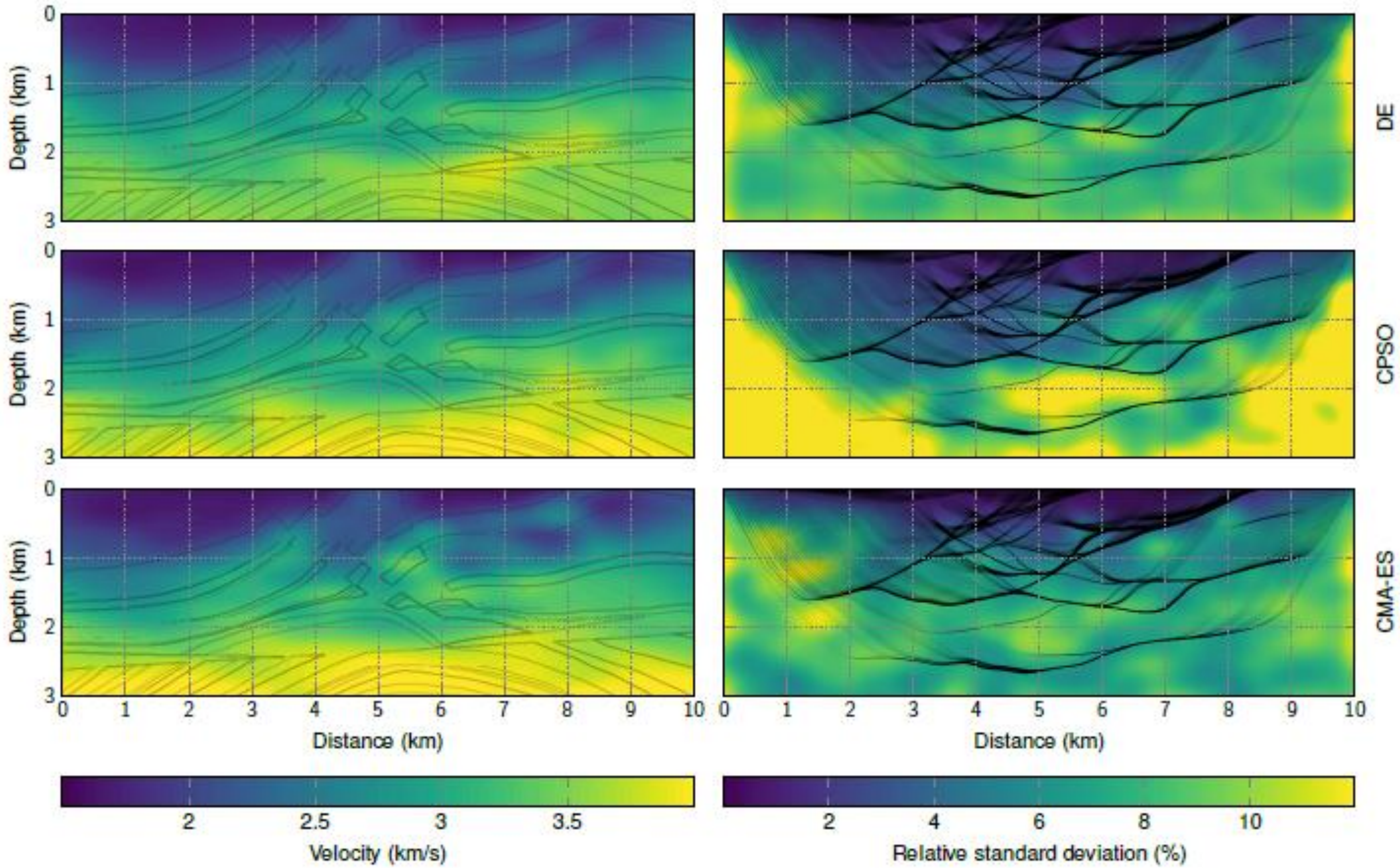


- Only 3 hours of computation time for 300 parameters

Luu et. al., submitted to Geophys. Prosp.

Competitive Particle Swarm Optimization

○Comparison of 3 evolutionary algorithms



- Uncertainties consistent with ray coverage for CPSO

Luu et. al., submitted to Geophys. Prosp.

- Perspectives
  - Extension à 3D
  - Paramétrisation : représentation réaliste du modèle de vitesse
  - Prise en compte d'autres attributs de l'onde => Surrogate?